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TERRESTRIAL NEUTRINOS

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Abstract: Arguments are given for a remarkable abundance of radioactive elements within the earth. Methods are discussed in order to measure this abundance by neutrino experiments.

1. Introduction

The chemical composition of the interior of the earth is still quite unsure. Therefore it would be an advancement if one could measure the abundance of selected chemical elements by their neutrino emission. In this connection, potassium, thorium and uranium are of interest. Generally it is assumed that these elements are confined mainly to the earth's crust, if only for that reason that the material obtainable from the earth's mantle shows a small portion of the mentioned substances. On the other hand the exchange of material between the earth's crust and the upper mantle can give rise to a reduction of radioactive elements in this region. Further there are arguments from paleomagnetic measurements [1,2]) and from geophysical considerations [3]) for an increase of the earth's radius. This would mean that within the earth there is sufficient radioactive material in order to blow up our planet. The geophysical argument is based on the fact that one year had 377 solar days 135 million years ago and had 371 solar days 65 million years ago [4]). Thus at present the period of the earth's rotation increases by 2.11 msec in one century. The present rotation energy of the earth amounts 2.158×10^{36} erg and the energy loss by the tides [5]) is approximately given by 1.7×10^{19} erg/sec. From these numbers we get a mean increase of the earth's radius of 0.077 cm in one year. The mean gravitation energy of the mass unit amounts -4.16×10^{11} erg/g. Thus the growth of the radius means an increase of the gravitation energy by 1.60×10^{-6} erg/g · sec = 1.20 μcal/g · y. If we add 0.18 cal/g · y for the mean increase of the thermal energy, then the mass unit within the earth needs an energy supply of 1.38 μcal/g · y from the decay of radioactive elements. We assume a mass ratio of 11.3 mgK : 3.4 gTh : 1 gU. (This ratio is characteristic for the earth's crust.) The energy production of these elements is given by

$$27\ \mu\text{cal/gK} \cdot \text{y}, \qquad 0.20\ \text{cal/gTh} \cdot \text{y}, \qquad 0.71\ \text{cal/gU} \cdot \text{y}.$$

Therefore the concentrations of potassium, thorium and uranium become on the average

$$9.2\ \text{mgK/g}, \qquad 2.8\ \mu\text{gTh/g}, \qquad 0.82\ \mu\text{gU/g}. \tag{1}$$

These abundances are roughly the same as in basalts [6]) and are much higher than one assumes usually.

If we follow the development of the earth's dimensions in the past, we find about 3.8×10^9 y ago a surface of 1.7×10^{18} cm^2. At that time the surface of the earth could have been covered totally with continental blocks. The volume of the oceans would have been formed by and by during the history of the earth. As there are early sedimentation processes we also should assume that the present water volume of 1.37×10^{24} cm^3 had not existed from the earliest beginnings, but has been produced gradually and mainly by conversion of serpentine into olivine and water.

The increase of the temperature within the upper mantle of the earth gives rise to this process, which seems to play an important role for the continental drift [7]). Under the assumption of a uniform water production within the past 3.8×10^9 y, we get a production rate of 1.1×10^7 g water/sec or 45000 m^3/sec water vapour at a temperature of 600° C.

We have seen that the high abundance (1) of radioactive materials has important consequences for the chemical composition of the earth and for the dynamics of the earth during its history. Therefore it is desirable to measure the radioactivity of the earth. In sect. 2 we discuss the antineutrino and neutrino flux produced by the earth. In sect. 3 we consider the two nuclear reactions

$$\nu + {}^{37}\mathrm{Cl} \rightarrow \mathrm{e}^- + {}^{37}\mathrm{Ar} - 0.816 \text{ MeV},$$
$$\bar{\nu} + \mathrm{p} \rightarrow \mathrm{e}^+ + \mathrm{n} - 1.804 \text{ MeV}. \qquad (2)$$

in order to determine the production rates for negative and positive electrons.

2. The Neutrinos from Potassium, Thorium and Uranium

Potassium produces neutrinos (ν), antineutrinos ($\bar{\nu}$) and electrons (e^-). Natural potassium contains the radioactive isotope ^{40}K in 0.0118% abundance. This isotope decays by two modes

$${}^{40}\mathrm{K} + \mathrm{e}^- \rightarrow {}^{40}\mathrm{Ar} + \nu + 1.513 \text{ MeV } (11\,\%),$$
$${}^{40}\mathrm{K} \rightarrow {}^{40}\mathrm{Ca} + \mathrm{e}^- + \bar{\nu} + 1.321 \text{ MeV } (89\,\%), \qquad (3)$$

with a half life

$$t_0(\mathrm{K}) = 1.27 \times 10^9 \text{ y}.$$

Therefore the neutrino and the antineutrino production of potassium is given by

$$\begin{aligned}(6.0225 \times 10^{23} \times 1.18 \times 10^{-4} \times 0.11\nu \times 0.69315)/(1.27 \times 10^9 \\ \times 3.1557 \times 10^7 \text{ sec} \times 39.102 \text{ gK}) \\ = 0.11\nu \times 31.4/\text{gK} \cdot \text{sec} = 3.46\nu/\text{gK} \cdot \text{sec},\end{aligned}$$

$$0.89\bar{\nu} \times 31.4/\text{gK} \cdot \text{sec} = 28.0\bar{\nu}/\text{gK} \cdot \text{sec}.$$

The neutrinos are monochromatic ($E = 1.513$ MeV); the antineutrinos have energies between 0 and 1.321 MeV.

Thorium decays with a half-life of

$$t_0(\mathrm{Th}) = 1.39 \times 10^{10}\ \mathrm{y}.$$

For radioactive equilibrium we get the reaction

$$^{232}\mathrm{Th} \rightarrow {}^{208}\mathrm{Pb} + 6\,{}^{4}\mathrm{He} + 4\bar{\nu} + 42.8\ \mathrm{MeV}, \tag{4}$$

if we take together all daughter nuclei of the Th decay. Thus the antineutrino production becomes

$$6.0225 \times 10^{23} \times 4\bar{\nu} \times 0.69315/1.39 \times 3.1557 \times 10^{17}\ \mathrm{sec} \times 232.038\ \mathrm{gTh}$$
$$= 1.64 \times 10^{4}\ \bar{\nu}/\mathrm{gTh} \cdot \mathrm{sec}.$$

Uranium has a half-life of

$$t_0(^{238}\mathrm{U}) = 4.51 \times 10^{9}\ \mathrm{y}.$$

The decay of uranium

$$^{238}\mathrm{U} \rightarrow {}^{206}\mathrm{Pb} + 8\,{}^{4}\mathrm{He} + 6\bar{\nu} + 51.7\ \mathrm{MeV} \tag{5}$$

gives rise to an antineutrino current of

$$6.0225 \times 10^{23} \times 6\bar{\nu} \times 0.69315/4.51 \times 3.1557 \times 10^{16}\ \mathrm{sec} \times 238.03\ \mathrm{gU}$$
$$= 7.39 \times 10^{4}\ \bar{\nu}/\mathrm{gU} \cdot \mathrm{sec}.$$

Now we want to estimate the contributions of the earth's crust and of the whole earth to the neutrino flux. Within the crust we find on the average

$$29.7\ \mathrm{mgK/g}, \qquad 9.0\ \mu\mathrm{gTh/g}, \qquad 2.6\ \mu\mathrm{gU/g}. \tag{6}$$

This means an energy production of 1.4×10^{-13} cal/g · sec. The heat emission per unit area amounts [8])

$$63.9\ \mathrm{erg/cm^2 \cdot sec} = 1.53\ \mu\mathrm{cal/cm^2 \cdot sec}. \tag{7}$$

Eqs. (6) and (7) give a mass ρ per unit area

$$\rho(\mathrm{crust}) = 1.08 \times 10^{7}\ \mathrm{g/cm^2}, \qquad \rho(\mathrm{Th}) = 97\ \mathrm{gTh/cm^2},$$
$$\rho(\mathrm{K}) = 3.21 \times 10^{5}\ \mathrm{gK/cm}, \qquad \rho(\mathrm{U}) = 28\ \mathrm{gU/cm^2}, \tag{8}$$

for the continents. If we assume an area of 1.6×10^{18} cm^2 for the continental blocks and if we use the number 5.10×10^{18} cm^2 for the total surface of the earth, then the density j_c of the neutrino current from the earth's crust becomes

$$\begin{aligned}
j_c(\nu, \mathrm{K}) &= (1.6/5.1 \times 2)3.21 \times 10^{5} \times 3.46\nu/\mathrm{cm^2} \cdot \mathrm{sec} = 1.74 \times 10^{5}\nu/\mathrm{cm^2} \cdot \mathrm{sec},\\
j_c(\bar{\nu}, \mathrm{K}) &= (1.6/5.1 \times 2)3.21 \times 10^{5} \times 28.0\ \bar{\nu}/\mathrm{cm^2} \cdot \mathrm{sec} = 14.1 \times 10^{5}\bar{\nu}/\mathrm{cm^2} \cdot \mathrm{sec},\\
j_c(\bar{\nu}, \mathrm{Th}) &= (1.6/5.1 \times 2)1.64 \times 10^{4} \times 97\ \bar{\nu}/\mathrm{cm^2} \cdot \mathrm{sec} = 2.6 \times 10^{5}\bar{\nu}/\mathrm{cm^2} \cdot \mathrm{sec},\\
j_c(\bar{\nu}, \mathrm{U}) &= (1.6/5.1 \times 2)7.4 \times 10^{4} \times 28\ \bar{\nu}/\mathrm{cm^2} \cdot \mathrm{sec} = 3.3 \times 10^{5}\bar{\nu}/\mathrm{cm^2} \cdot \mathrm{sec}.
\end{aligned} \tag{9}$$

Therefore the total density of the neutrino currents from the earth's crust amount is

$$\begin{aligned} j_c(\nu) &= j_c(\nu, \mathrm{K}) = 1.74\times 10^5 \nu/\mathrm{cm}^2 \cdot \mathrm{sec}, \\ j_c(\bar{\nu}) &= \qquad\quad 20.0\times 10^5 \bar{\nu}/\mathrm{cm}^2 \cdot \mathrm{sec}. \end{aligned} \tag{10}$$

For the abundances [1]) of potassium, thorium and uranium within the remaining part of the earth, we find the current densities

$$\begin{aligned} j_r(\nu, \mathrm{K}) &= 0.561\times 10^8 \nu/\mathrm{cm}^2 \cdot \mathrm{sec}, \\ j_r(\bar{\nu}, \mathrm{K}) &= 4.54 \ \times 10^8 \bar{\nu}/\mathrm{cm}^2 \cdot \mathrm{sec}, \\ j_r(\bar{\nu}, \mathrm{Th}) &= 0.801\times 10^8 \bar{\nu}/\mathrm{cm}^2 \cdot \mathrm{sec}, \\ j_r(\bar{\nu}, \mathrm{U}) &= 1.06 \ \times 10^8 \bar{\nu}/\mathrm{cm}^2 \cdot \mathrm{sec}. \end{aligned} \tag{11}$$

The total contribution of the remaining part of the earth becomes

$$j_r(\nu) = 0.561\times 10^8 \nu/\mathrm{cm}^2 \cdot \mathrm{sec}; \qquad j_r(\bar{\nu}) = 6.40\times 10^8 \bar{\nu}/\mathrm{cm}^2 \cdot \mathrm{sec}. \tag{12}$$

For the current densities from the whole earth, eqs. (9) and (11) yield

$$\begin{aligned} j(\nu, \mathrm{K}) &= 0.563\times 10^8 \nu/\mathrm{cm}^2 \cdot \mathrm{sec}, \\ j(\bar{\nu}, \mathrm{K}) &= 4.55 \ \times 10^8 \bar{\nu}/\mathrm{cm}^2 \cdot \mathrm{sec}, \\ j(\bar{\nu}, \mathrm{Th}) &= 0.804\times 10^8 \bar{\nu}/\mathrm{cm}^2 \cdot \mathrm{sec}, \\ j(\bar{\nu}, \mathrm{U}) &= 10.7 \ \times 10^8 \bar{\nu}/\mathrm{cm}^2 \cdot \mathrm{sec}. \end{aligned} \tag{13}$$

Finally the sum of the current densities from the whole earth becomes

$$j(\nu) = 0.563\times 10^8 \nu/\mathrm{cm}^2 \cdot \mathrm{sec}; \qquad j(\bar{\nu}) = 6.42\times 10^8 \bar{\nu}/\mathrm{cm}^2 \cdot \mathrm{sec}. \tag{14}$$

3. Detectable Neutrinos

Only a small part of the terrestrial neutrinos is measurable. We expect a neutrino current density of $0.56\times 10^8 \nu/\mathrm{cm}^2$ sec from the decay of potassium within the whole earth.

The neutrinos have an energy of 1.513 MeV. They give rise to the nuclear reaction

$$\nu + {}^{37}\mathrm{Cl} \rightarrow \mathrm{e}^- + {}^{37}\mathrm{Ar} - 0.816 \ \mathrm{MeV}. \tag{15}$$

This reaction is used to measure the current of solar neutrinos [9,10]). The current density of the solar neutrinos amounts $(0.67\pm 0.01)\times 10^{11}$ $\nu/\mathrm{cm}^2 \cdot$ sec. We assume that 4 out 100 He atoms of the sun are produced by the Weizsäcker-Bethe cycle (C-N-cycle).

Then the decays of ^{13}N and ^{15}O contribute with 1.1×10^9 $\nu/\mathrm{cm}^2 \cdot$ sec to the reaction (15). This means that there are only 4 terrestrial neutrinos in 100 neutrinos of the total radiation which can be measured by the process (15). Solar and terrestrial neutrinos can only be distinguished from each other by the fact that the terrestrial neutrinos produce monochromatic electrons with a kinetic energy of 0.697 MeV, whereas the electrons produced by solar neutrinos have continuous energies between 0 MeV and 0.923 MeV. (Only for neutrinos from the ^{8}B decay can the electrons reach an energy of 13 MeV; but these electrons are not so abundant as the electrons of low energy).

For terrestrial neutrinos the cross section of ^{37}Cl in the reaction (15) becomes 1.4×10^{-44} cm^2. Therefore we find only one electron of 0.697 MeV per year within a target of 10 tons natural chlorine. On account of the small production rate it is difficult to determine the abundance of potassium within the earth.

For the antineutrinos from the decay of thorium and uranium the situation is somewhat better. We consider the inverse process to the decay of a neutron n into a proton p and an antineutrino

$$\bar{\nu}+\mathrm{p} \rightarrow \mathrm{e}^{+}+\mathrm{n}-1.804 \text{ MeV}. \tag{16}$$

From the decays of ^{212}Bi (ThC), ^{234}Pa (UX$_2$), ^{214}Bi (RaC) and ^{210}Tl (RaC″), we get antineutrinos with energies $E > 1.804$ MeV. The decay of the neutron (ft = 1198 sec) can be used to calculate the cross section for the process (16)

$$\begin{aligned}
&\sigma(\bar{\nu}+p \rightarrow \mathrm{e}^{+}+\mathrm{n}) \\
&= (2\pi^2\hbar^3 \log 2/m_e^5 c^8 ft)(E-1.293 \text{ MeV})[(E-1.293 \text{ MeV})^2-0.261127 \text{ (MeV)}^2]^{\frac{1}{2}} \\
&= 0.8397\times10^{-43}\text{ cm}^2(E/\text{MeV}-1.293)[(E/\text{MeV}-1.293)^2-0.261127]^{\frac{1}{2}}. \qquad (17)
\end{aligned}$$

TABLE 1

The radiation caused by terrestrial antineutrinos

A	El	E_m	ε_1	ε_2	σ_m	α
210	Tl	0.16	0.38	0.81	0.24	5×10^{-3}
212	Bi	0.45	0.33	1.14	0.65	9
214	Bi	1.38	0.29	2.11	2.88	80
234	Pa	0.53	0.32	1.23	0.79	11

The source of the antineutrinos has the mass number A (element El). The positive electrons have the maximum energy E_m (MeV). The corresponding cross section amounts $\sigma_m\times10^{-43}$ cm^2. The energy of the γ-quanta from the e$^+$-annihilation ranges from ε_1 MeV to ε_2 MeV. The abundance α of the e$^+$-radiation is given in per cent.

The maximum energies E_m of the positive electrons and the maxima σ_m of the corresponding cross sections for the process (16) are given in table 1. From eqs. (13) and (17) we find the total number of positive electrons produced by one proton

$$1.4\times10^{-36}\ \mathrm{e}^{+}/\mathrm{sec}\cdot\text{proton}.$$

This means that we can get four positive electrons or four pairs of γ-quanta per day from 500 tons water. There are no solar antineutrinos which would disturb the measurements.

4. Conclusion

Paleomagnetic measurements and geophysical considerations make it probable that on the average the radius of the earth increases by 0.8 mm per year. If this expansion is due to the energy production of radioactive materials within the earth, then we expect a drastic enlargement of the surface of the earth during its history. Further the mean abundance of potassium, thorium and uranium must be in the same order of magnitude as that for basalts. The electron capture of potassium (^{40}K) gives rise to a neutrino current density of 0.56×10^8 neutrinos/cm^2 · sec, which is able to produce one electron of 0.7 MeV per year within a target of 10 tons natural chlorine. Only by their well-defined energy can these electrons be distinguished from the electrons by solar neutrinos.

From the decay of potassium, thorium and uranium we expect an antineutrino current density of 64×10^7 antineutrinos/cm^2 · sec. But only 1.6×10^7 antineutrinos/cm^2 · sec have sufficient energy to transform a proton into a neutron. From the reaction

$$\bar{\nu} + p \rightarrow e^+ + n \rightarrow 2\gamma - e^- + n,$$

we expect four pairs of γ-quanta per day within a target of 500 tons water. (If there would be contributions only from the earth's crust, then we would get 4.5 pairs of γ-quanta per year). Most of the positive electrons and γ-quanta (80 %) are produced by the decay of ^{214}Bi (RaC). The energy of these quanta range from 0.3 MeV to 2.1 MeV. Thus in principle we have a method in order to measure the uranium content of the earth. For thorium and potassium such a measurement seems to be possible but much more difficult.

References

1) L. Egyed, Nature **197** (1963) 1059
2) D. van Hilten, Nature **200** (1963) 1277
3) G. Eder, Z. Geophys. **31** (1965) 206
4) J. W. Wells, Nature **197** (1963) 948
5) A. Defant, Handbuch der Physik, Bd. **48** (Springer Verlag, Berlin, 1957) p. 926
6) S. Heier and J. J. Rogers, Geochim. Cosmochim. Acta **27** (1963) 137
7) E. Orowan, Science **146** (1964) 1003
8) G. J. Wasserburg, C. J. F. MacDonald, F. Hoyle and W. A. Fowler, Science **143** (1964) 465
9) J. N. Bahcall, Phys. Rev. Lett. **12** (1964) 300
10) R. Davis, Phys. Rev. Lett. **12** (1964) 303